An Overview of Switched Systems Leveraging Networked Control Systems

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Abstract

This paper provides a concise overview of the evolution of Networked Control Systems (NCS) and switched systems, combines network control with switched control to analyze and study their structure and characteristics, describes some problems faced by networked switched systems, summarizes the corresponding solutions, and further prospects the future of network-based switched systems.

Keywords: NSCS, Lyapunov Function, Dwell time

1. Introduction

Network technology has been widely applied globally and has penetrated all aspects of society. In the field of control, the introduction of networks not only leads to a quiet revolution in control structures but also heralds innovations in control modes. Compared to traditional point-to-point control structures, the advantages of this network-linked control framework are evident: low cost, low power consumption, easy installation and maintenance, resource sharing, and remote operation. These advantages have led to its increasing application in fields such as telemedicine, intelligent transportation, aerospace, smart manufacturing, and national defense. In recent years, research on networked switched systems has garnered joint attention from researchers in control, signal processing, and communication fields, with a plethora of related literature emerging.

As modern industrial control systems become increasingly complex, the demand for mobility of control nodes grows, and the geographical distribution of system components expands, the integration of communication networks into control systems is receiving deeper attention and advancement. Since the late 1980s, when researchers explored integrated communication control systems, the term "networked control systems" was later proposed. Over the past two decades, various network control frameworks and methods have proliferated, greatly enriching this new control theory. The ART Laboratory at the University of Alberta, Canada, has studied the impact of packet loss and delays on remote control systems and achieved certain results. Scholars have provided detailed analyses and discussions of the performance of networked control systems. Researchers have proposed various control methods such as optimal planning and optimal control for remote networked control robots, achieving notable results.

However, networks are not a reliable communication medium. Due to the physical limitations of bandwidth and service capacity, issues such as delays, packet loss, and timing disorders are inevitable during data transmission. These problems are significant factors that degrade system performance and destabilize such artificial systems. Moreover, their presence makes it difficult to directly apply traditional control theory to the analysis and design of such systems, especially for hybrid switched control systems with nonlinearity and strong coupling. To ensure the stability and satisfactory control performance of such systems, it is essential to deeply study networked control systems, particularly the deep integration of networks and switched control systems, and explore their development and related analysis and design theories. Some researchers have proposed using hybrid system theory to address networked control issues, with switched systems, as an important branch of hybrid systems, playing a crucial role in solving networked control problems.

From the perspective of control systems, switching mechanisms introduce additional degrees of freedom in controller design. Switching can be artificially introduced to model highly complex nonlinear systems and large uncertain systems. Researchers studied the stability of linear switched systems, using quadratic Lyapunov-like functions as tools to provide sufficient conditions for asymptotic stability. Other scholars discussed the consensus problem of dynamic agents with fixed networks and switching topologies. In 1999, researchers described three issues related to the stability of switched systems and the design of switching laws: stability under arbitrary switching sequences, stability under certain useful classes of switching sequences, and the design of stable switching sequences, marking the substantive development of switched system research.

Studies have examined a class of nonlinear switched control systems with time-varying network delays. Other research investigated the stability of singularly perturbed continuous-time switched linear systems, pointing out that under arbitrary switching rules, the corresponding two-time-scale switched system satisfies stability conditions for both slow and fast switching subsystems. State feedback control laws based on LMI conditions were provided to stabilize continuous-time linear switched systems. Additional studies have explored the stability of discrete-time switched systems with uncertain switching signals, noting that uncertain switching signals can destabilize switched systems and providing stability conditions for such systems. This method is applicable to networked switched control systems with time-varying delay characteristics.

Due to the widespread application of switched systems in practical control systems, they can be used to model systems affected by known or unknown parameter variations, such as synchronous linear switched systems or periodically varying networked switched systems. Such systems naturally arise in the study of multi-rate sampling systems, attracting increasing attention to the research of networked switched systems.

Research has studied the switching between current control and voltage control modes to achieve grid-connected/islanded switching, demonstrating the application of switched control in power grid systems. Other studies used phase plane control and quasi-sliding mode switching control methods to propose angle feedback switching control laws to enhance the robustness of attitude control systems. Additional work addressed linear time-invariant systems with multiple subsystems, using average dwell time and multiple Lyapunov function methods to improve overall disturbance attenuation performance.

This paper aims to review important achievements in this field in recent years, summarize them, and point out future research directions and unresolved new challenges.

2. Problem Description

2.1 Basic Issues in Networked Control Systems

On one hand, networked control systems offer significant advantages such as low cost, flexible control system design, easy installation and maintenance, reliable information exchange, and successful industrial applications like remote-controlled robots, unmanned combat command systems, remote surgery, and intelligent buildings and transportation. On the other hand, introducing networks into feedback loops also brings negative effects.

First, due to limited network bandwidth, simultaneous data transmission by multiple nodes can cause delays, degrading system performance or even leading to instability. Second, various uncertain interferences during network transmission can result in data errors or loss, known as packet loss. Based on the causes, packet loss can be categorized into three types:

a) Loss caused by different transmission path characteristics or network faults.

b) Loss caused by multiple nodes competing for limited network resources.

c) Proactive discarding of non-critical or secondary packets to improve network load conditions and prevent system deterioration.

Third, a single packet can only carry limited information. For large data volumes, data must be split into multiple packets for transmission, known as multi-packet transmission. Fourth, in a network environment, transmitted data passes through numerous computers and communication devices with non-unique paths, inevitably leading to timing disorders. This refers to the situation where multiple complete data packets with a certain original sequence arrive at the target node in a different order than the original sequence.

2.2 Issues in Switched Systems

From both theoretical and practical perspectives, studying switched systems is beneficial, but it also presents many challenges. For example, since instability can arise from switching between asymptotically stable systems or using switching to stabilize unstable systems, the switched control of multiple linear time-invariant (LTI) devices has been recognized as a significant problem. Based on this, existing research primarily addresses the following two major directions:

2.2.1 Conditions to Ensure Asymptotic Stability of a Switched System Under Arbitrary Switching Signals

Under arbitrary switching, the stability conditions for a switched system composed of multiple stable subsystems are provided by a common Lyapunov function. Therefore, finding a suitable Lyapunov function becomes the focus of research on the stability of arbitrarily switched systems. Studies have shown that the existence of a common Lyapunov function provides stability conditions for switched systems composed of multiple stable subsystems. The existence of arbitrary switching compensators has been discussed in the literature. Using the Lyapunov function method, sufficient conditions to ensure the asymptotic stability of switched systems described by output and difference equations have been provided. Based on this, a stabilizing controller design method that is not restricted by switching conditions but can asymptotically stabilize the switched system has been proposed.

The use of digital methods has also played an important role in studying the stability of systems under arbitrary switching. Researchers employed Lie algebra theory to transform the problem of the existence of a common first-order Lyapunov function under arbitrary switching strategies into the solvability of Lie algebra generated by system matrices. Conditions for the existence of a common Lyapunov function for a pair of non-commutative systems have been proposed. Other studies presented sufficient conditions under which Lyapunov inequalities with different coefficient matrices can have common solutions. Inverse Lyapunov theorems indicate that quasi-quadratic or piecewise quadratic common functions can be used for linear switched systems that are asymptotically stable under arbitrary switching signals.

2.2.2 Stability of Switched Systems Under Restricted Switching Operations and the Design of Switched Systems

Assuming each subsystem is stable, the characteristics of the switched system can be ensured under sufficiently slow switching. Even if some subsystems are unstable, as long as these unstable subsystems are activated for relatively short periods, stable switching laws can still be obtained, such as the average dwell time scheme. Research on this problem mostly employs the multiple Lyapunov function method. The average dwell time method has been combined with the multiple Lyapunov function method to study the weighted disturbance attenuation problem of linear switched systems for two scenarios: all subsystems being stable and not all subsystems being stable.

A method based on verifiable conditions for the existence of a common bi-positive linear Lyapunov function for a pair of positive LTI systems has been proposed to determine whether a given switched positive continuous-time linear system is exponentially stable. Future work will extend these results to arbitrary finite sets, such as LTI systems, and develop synthesis procedures to utilize these results for designing stable positive systems. Additionally, explanations have been provided from perspectives such as average time switching, sliding mode control, and finite-time switching control.

For a family of signals, the problem can be reduced to finding a common quadratic Lyapunov function for an uncontrolled system under a specific "R" space. Matrix constraint conditions have been used to provide necessary and sufficient conditions for the existence of a common quadratic function for linear switched systems with only two subsystems. However, this approach cannot handle cases where all subsystems are unstable.

The switching mode problem for discrete-time linear switched systems has been discussed. Based on the abstraction-aggregation method, a state feedback path-switching method has been proposed. Research has proven that the state feedback path-switching law is universal in the sense that any stabilizing switching law for switched linear systems admits this method. The proposed switching law generates universal switching signals for stability problems, but this method is difficult to apply to performance criteria and control performance issues.

Linear matrix inequality (LMI) solutions have been used to find Lyapunov functions and design feedback control. Studies have examined the relationship between asymptotic stabilizability and output stabilizability for linear switched systems under the condition that each subsystem is controllable and observable, and have designed observer-based stabilizable controllers. Counterexamples have been provided to show that linear switched systems with control switches can be stabilized by appropriate switching laws without necessarily providing convex Lyapunov functions.

In other research, switching devices under stable and quadratically stable switched linear compensators have been given necessary and sufficient stabilization conditions. If these conditions are met, regardless of how a family of compensators is associated with a family of switching devices, there exists a real quantity such that the loop system switching is stable. The literature has also demonstrated how to achieve these realizations.

2.3 Issues in Networked Switched Systems with Actuator Saturation

Stability analysis and control synthesis conditions for switched control have been addressed using bilinear matrix inequalities (BMIs). However, this method cannot guarantee system convergence or even achieve local optimal solutions. Based on this, some scholars have begun to study and develop design frameworks for switched control to solve important practical problems such as constraints, nonlinear saturation, and actuator saturation.

2.3.1 Analysis and Control of Switched Linear Systems with Actuator Saturation

As a state-dependent switching strategy, the minimal switching logic was mentioned in earlier research. This strategy does not require each subsystem to be stable and includes a special case of quadratic stability conditions. Through this method, the design of state feedback and dynamic output feedback H-infinity controllers has been further developed. Recent research results have proven that the minimal switching strategy can guarantee better H2 performance than that produced by a single subsystem. This makes it a feasible method not only for all unstable subsystems in switched systems but also for some or all stable subsystems. Additionally, the minimal switching rule has been relaxed in the literature to avoid further effects caused by high-frequency switching.

On the other hand, actuator saturation is an inherent nonlinearity in all dynamic systems. The problem of actuator saturation has been extensively studied in the academic community, including topics such as stability, output regulation, and disturbance rejection. In recent years, the focus has shifted to exponentially unstable open-loop systems, as many control systems with actuator saturation, such as flight control, are exponentially unstable. Since only part of the state space of systems with actuator saturation is zero-controllable, the goal is to characterize the zero-controllable region and design feedback laws that allow the system to operate over the entire or most of the zero-controllable region. Different frameworks have proposed various design procedures based on rigorous theoretical analysis. However, most literature on exponentially unstable systems focuses on state feedback, with very few addressing output feedback.

2.3.2 Analysis and Switching Control Problems for Continuous/Discrete Switched Linear Systems

All observer-based stabilizing controllers are parameterized under the assumption that the device configuration has exact knowledge. However, this method for ensuring stability is too conservative, especially under specific switching rules. Since many systems in practice cannot be controlled by a common Lyapunov function, researchers have turned their attention to stability under some correctly chosen switching logics, and multiple Lyapunov functions have proven to be very useful tools for stability analysis. Therefore, the focus has shifted to using multiple Lyapunov functions as tools for controlling switched systems.

For this purpose, various switching rules have been proposed for switched systems, which can generally be divided into state-dependent and time-driven categories. Research has discussed a system composed of two unstable linear autonomous state models. Control is achieved by selecting which system to activate. The control goal is the quadratic stability of state trajectories through switching between two unstable systems. Assuming there exists a stable combination of two unstable system matrices, three stable control strategies have been discussed: time-averaged control, variable structure control, and a hybrid feedback control.

Other studies have discussed control laws for stable linear multi-mode systems, selecting a specific system mode. The considered systems are similar to jump linear systems, where the active mode is selected by a stochastic process. Here, the controller uses a piecewise quadratic Lyapunov-like function to select the system mode to activate. The paper addresses the existence of solutions to coupled Lyapunov equations for stable control laws. The solutions to these equations are discussed in relation to the eigenvalue locations of certain matrix sub-blocks from the component system matrices and coupling parameters. These conditions do not require every subsystem to be stable and include a special condition—the quadratic stability condition.

The underlying minimal switching strategy is a special method that satisfies the difficult-to-check non-increasing condition of multiple Lyapunov functions and is easy to implement. A special switching strategy meets the non-increasing condition of multiple Lyapunov functions, and the minimal switching strategy can be chosen. However, this minimal switching has the potential drawback of chattering phenomena.

2.3.3 Analysis and Control of Switched Linear Systems Through Minimal Switching and Dwell Time

Assuming each subsystem is stable, the characteristics of the switched system can be ensured under sufficiently slow switching, i.e., using the average dwell time scheme. Even if some subsystems are unstable, as long as these unstable subsystems are activated for relatively short periods, stable switching laws can still be obtained. Research has demonstrated the construction of Lyapunov functions for switched systems. This method requires finding stable combinations of system matrices. The average dwell time method has been combined with the multiple Lyapunov function method to study the weighted disturbance attenuation problem of linear switched systems for two scenarios: all subsystems being stable and not all subsystems being stable.

Research on linear switched unstable systems under dwell time constraints is still lacking. The concept of average dwell time in stable switched systems is based on slow switching that allows for brief oscillations during switching. In other words, the decrease in the Lyapunov function during dwell time compensates for the potential increase in the Lyapunov function at switching instances. The piecewise function is represented by $v_x(x)$, and the known condition to limit the increase in the Lyapunov function is $v_x(x) \leq \mu v_j(x)$, when $x > 1$ are indices of the system before and after switching. When piecewise quadratic Lyapunov functions are adopted, the condition can be simplified to $P_i \leq P_j \leq \mu P_j$. If possible, the decrement of the Lyapunov function at switching instances can be used to compensate for the potential increase in the Lyapunov function during dwell time.

To address the analysis and control of switched linear systems with unstable subsystems under dwell time constraints, dwell time can be combined with a modified minimal switching logic to propose a new switching strategy.

3. Research Methods

3.1 Norm-Bounded Differential Inclusion (NDI) Method

A general framework can be established to analyze and design control for switched linear systems with actuator saturation. Two types of linear differential inclusions (LDI) describe saturated switched systems: polyhedral differential inclusions (PDI) and norm-bounded differential inclusions (NDI). Combining a state-dependent minimal switching strategy, stability analysis and controller synthesis conditions can be derived. A set of switched output feedback controllers is designed to minimize the disturbance attenuation level defined by a class of energy-bounded disturbance gain regions. Academic studies have shown that the norm-bounded differential inclusion (NDI) method is superior. Compared to those obtained based on PDI, the increase in control input quantity is more scalable. This is based on a comparison of computational costs in terms of the number of optimization variables and matrix inequalities.

Using polyhedral differential inclusion (PDI) to describe actuator saturation, an output feedback law is designed to stabilize the saturated system over a large domain of attraction. Gain scheduling and the form of the output feedback controller have been implemented in prior research, achieving local stability and reducing the impact of disturbances on system output. Therefore, control synthesis conditions are formulated in the form of linear matrix inequalities (LMIs). Using norm-bounded differential inclusion (NDI) to describe actuator saturation, an internal dead-zone loop output feedback controller is proposed as a saturated control synthesis method. The synthesis conditions for this controller can be transformed into LMI forms.

3.2 Lyapunov-Metzler Inequality Method

The state-dependent minimal switching strategy is enhanced to mitigate its inherent chattering behavior. Whenever a different subsystem attains the minimal Lyapunov function, a relaxed minimal switching strategy is based on switching evolution rather than being forced, but it is maintained until the Lyapunov function's minimal value is less than that of the active subsystem's Lyapunov function by a certain margin. A special class of matrix inequalities, known as Lyapunov-Metzler inequalities, is modified under a relaxed minimal switching strategy to provide conditions for output feedback control synthesis and stability analysis.

The switching rule, combined with the design of switched output feedback controllers, ensures the stability of the switched system and satisfies a pre-specified secondary gain performance. The proposed analysis and switching control methods can avoid frequent switching, which is typically adhered to in designs based on minimal switching. The Lyapunov-Metzler inequality is generalized to include its original state as a special case. Research has discussed the stability of continuous-time switched linear systems under two strategies. The first is open-loop in nature (trajectory-independent), based on determining the minimal dwell time for a family of quadratic Lyapunov functions. The key point in calculating dwell time is that the proposed stability conditions do not require the Lyapunov function to decrease uniformly at every switching instance.

The second is closed-loop in nature (path-dependent), and the design solution provides stability conditions represented by Lyapunov-Metzler inequalities (including chattering). A non-convex and more conservative simplified version of the Lyapunov-Metzler inequality is offered. The relaxed minimal switching strategy can achieve stability in closed-loop switched systems under non-ideal disturbances or reduced performance. By using an adjustable parameter, the minimal switching strategy allows for a trade-off between performance reduction and chattering mitigation.

3.3 Improved Average Dwell Time Method

The analysis and switching control problems of switched linear systems are examined from different perspectives, leading to the proposal of a hybrid time-driven and state-dependent switching strategy. This strategy guarantees an average dwell time even when all subsystems are unstable. The switching rule and its associated switched output feedback controller design ensure the stability of the switched system and satisfy a pre-specified secondary gain performance.

Research has designed a state-dependent switching law subject to dwell time constraints, ensuring the stability of switched linear systems. Sufficient conditions for the stability of switched systems are obtained when the switching law is applied to various types of parameter uncertainties. A quadratic Lyapunov function is assigned to each subsystem, ensuring it is non-increasing at switching instances. During dwell time, this function changes piecewise linearly over time. The proposed method is also applicable to robust stabilization via state feedback. It is further extended to guarantee a bound on the secondary gain of the switched system and is used to derive state feedback control laws that achieve robustness under specified secondary gain constraints.

A new hybrid state-dependent and time-driven switching strategy is proposed for switched systems with all unstable subsystems. Unlike the well-known average dwell time method, which requires a decrease in the Lyapunov function at switching instances, this method ensures that some or all subsystems without stable performance have an average dwell time. By introducing a state-dependent switching rule, a solution is provided to guarantee a decrease in the Lyapunov function over certain state-space regions. This offers a more general framework for analyzing switched linear systems, where dwell time is not a special case, incorporating both minimal switching and modified minimal switching strategies as special methods.

3.4 Simulation Research on Networked Switched Control Systems

The stability of networked switched systems and the study of related control performance under network-induced factors require validation through simulation. Therefore, simulation experiments have become a hot topic in research. The Jitterbug toolbox, developed by Lund University based on Matlab, provides a platform for analyzing time delays in networked control systems. It can perform static performance prediction and analysis for linear closed-loop control systems under various time-delay conditions.

In Jitterbug, a networked control system is described by two parallel models: the signal model and the time model. The signal model consists of a series of linear, continuous/discrete-time systems, while the time model is composed of a series of time nodes.

Networked switched systems combine network transmission and switched systems into a complex system. Although the Jitterbug toolbox provides effective and convenient technical support for simulating networked switched systems, its applicability is limited. Therefore, the demand for simulation technologies and tools will become a major focus in the fields of control and computer research in the future.

4. Prospects

In summary, networked control and switched control systems have been extensively studied, but research on the integration of networks and switching requires further expansion.

Among the switching rules mentioned above, a tunable positive parameter can be selected to reduce chattering, and the results show a reduction in the number of switches compared to the nominal minimal switching. However, with the introduction of network factors such as induced delays or communication constraints, obtaining such a parameter becomes more challenging. In the future, how to obtain such an ideal parameter remains a topic for further research.

Unlike the well-known average dwell time method, the proposed method determines a maximum average dwell time but does not impose requirements on system stability. Additionally, the state-dependent switching rule ensures a decrease in the Lyapunov function at switching instances. This work also provides a more general framework for analyzing switched linear systems, as it includes minimal switching and modified minimal switching as special cases. Currently, implementing the state-dependent switching rule requires information about the device's state. With the introduction of network factors, researching switching rules and relaxing this limitation will be a future research topic.

Most current research focuses on linear time-invariant systems. However, in practice, many subsystems cannot be modeled as linear time-invariant systems. Therefore, it is necessary to extend switched control theory to other types of systems. For example, linear parameter-varying (LPV) systems are often used in aerospace and process control industries. Controllers suitable for specific parameter sub-regions can be designed, and switching between them can achieve better parameter performance. Additionally, constraints such as time delays can be incorporated into this theory.

Currently, most of the switched systems studied are deterministic, meaning the transitions between different modes are deterministic. If the transitions between different models are arbitrary, the problem can be transformed into an arbitrarily switched system.

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